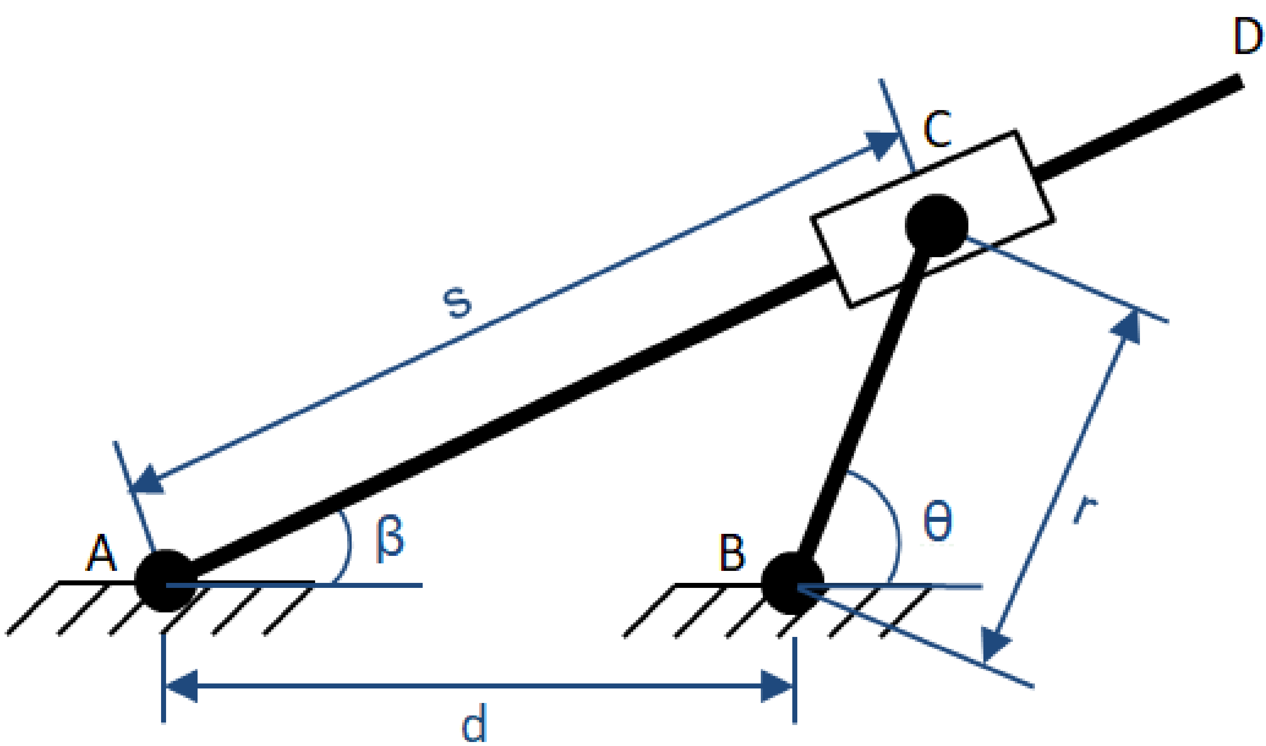
Kinematic Analysis of a Quick Return Device

Introduction

This is a quick return device.



This application will

- determine the range of motion of this device
- and its behavior if the crank driven at (i) a constant angular velocity, and (ii) a constant angular acceleration

The latter involves numerically solving differential equations. These are symbolically derived by differentiating the geometric relationships with respect to time. The resulting equations contain the first and second derivative of the crank angle with respect to time; these will be set to constant values to reveal the behavior of the system.

Parameters

> $restart:$

Distance between AB

> $d := 2.5:$

Crank length

> $r := 0.75:$

Angular velocity of crank for velocity analysis

> $\omega := 10:$

Angular acceleration of the crank for acceleration analysis

> $\alpha := 100:$

Displacement Analysis

These geometrical relationships may be written

> $eq1 := s\cos(\beta) = r\cos(\theta) + d:$
> $eq2 := s\sin(\beta) = r\sin(\theta):$

> $f := (theta\_val, s\_guess, beta\_guess) \rightarrow fsolve\left(\{eq1, eq2\}\Big|_{\theta = theta\_val}, \{s = s\_guess, \beta = beta\_guess\}\right):$

> $theta\_list := Vector\left(30, i \rightarrow \frac{2\cdot\pi}{30.}(i-1)\right):$
> $beta\_res := Vector(30):$
> $s\_res := Vector(37):$
> $init := f\left(0\,deg, r + d, \frac{5\cdot 2\cdot Pi}{180}\right):$
> $beta\_res_1 := rhs(select(has, init, \beta)[\,]):$
> $s\_res_1 := rhs(select(has, init, s)[\,]):$
> **for** i **from** 2 **to** 30 **do**
> $res := f\sim(theta\_list_i, s\_res_{i-1}, beta\_res_{i-1});$
> $beta\_res_i := rhs(select(has, res, \beta)[\,]);$

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    s_res[i] := rhs(select(has, res, s)[ ])
  end do:
> plot(theta_list, beta_res, labels = [theta, beta])
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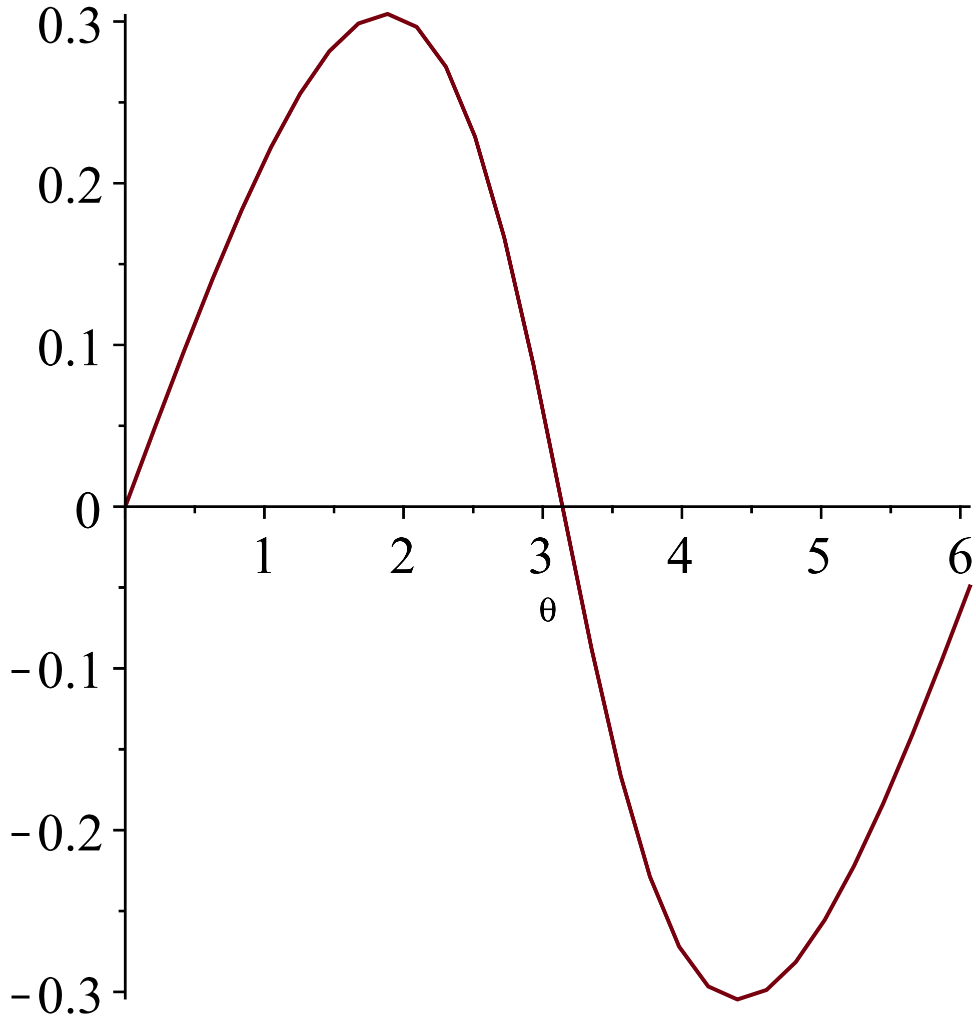

Velocity Analysis

Rewrite eq1 and eq2 as functions of time

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> eq1_t := subs({β = β(t), s = s(t), θ = θ(t)}, eq1);
  eq2_t := subs({β = β(t), s = s(t), θ = θ(t)}, eq2)
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$$eq1\_t := s(t)\cos(\beta(t)) = 0.75\cos(\theta(t)) + 2.5$$

$$eq2\_t := s(t)\sin(\beta(t)) = 0.75\sin(\theta(t)) \quad (4.1)$$

Differentiate eq1_t and eq2_t with respect to time. The resulting equations contain the derivative of the crank angle with respect to time - this will be set to a value of ω.

$$> de1 := \frac{\partial}{\partial t} eq1\_t = 0$$

$$de1 := \left(\left(\frac{d}{dt} s(t)\right)\cos(\beta(t)) - s(t)\left(\frac{d}{dt}\beta(t)\right)\sin(\beta(t)) = -0.75\left(\frac{d}{dt}\right.\right. \quad (4.2)$$

$\theta(t)\Big)\sin(\theta(t))\Big) = 0$

> $de2 := \frac{\partial}{\partial t}\, eq2\_t = 0$

$$de2 := \left(\left(\frac{d}{dt}\, s(t)\right)\sin(\beta(t)) + s(t)\left(\frac{d}{dt}\,\beta(t)\right)\cos(\beta(t)) = 0.75\left(\frac{d}{dt}\,\theta(t)\right)\cos(\theta(t))\right) = 0 \qquad \textbf{(4.3)}$$

Angular velocity of crank

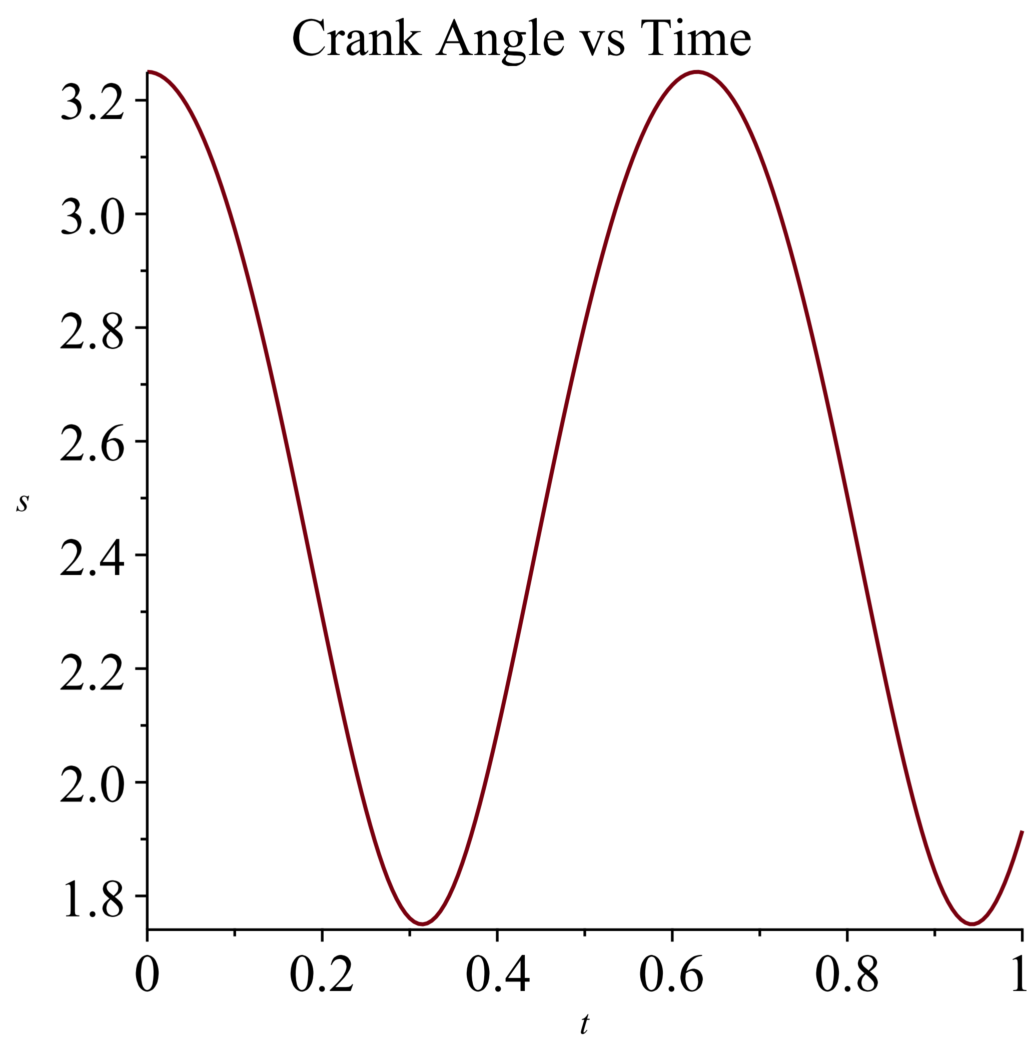
> $de3 := \frac{d}{dt}\,\theta(t) = \omega$

$$de3 := \frac{d}{dt}\,\theta(t) = 10 \qquad \textbf{(4.4)}$$

> $res2 := dsolve(\{de1, de2, de3, \beta(0) = 0, s(0) = 3.25, \theta(0) = 0\}, \{\beta(t), s(t), \theta(t), \}, numeric)$

$$res2 := \textbf{proc}(x\_rkf45)\ \ldots\ \textbf{end proc} \qquad \textbf{(4.5)}$$

> $plots{:-}odeplot(res2, [t, s(t)], t = 0..1, title = \text{"Crank Angle vs Time"})$



Acceleration Analysis

Differentiate de1 and de2 with respect to time. The resulting equations contain the second derivative of the crank angle with respect to time - this will be set to a value of α.

> $de4 := \frac{\partial}{\partial t}\, de1$

$$de4 := \left(\left(\frac{d^2}{dt^2}\, s(t)\right)\cos(\beta(t)) - 2\left(\frac{d}{dt}\, s(t)\right)\left(\frac{d}{dt}\, \beta(t)\right)\sin(\beta(t)) - s(t)\left(\frac{d^2}{dt^2}\, \beta(t)\right)\sin(\beta(t)) - s(t)\left(\frac{d}{dt}\, \beta(t)\right)^2\cos(\beta(t)) = -0.75\left(\frac{d^2}{dt^2}\, \theta(t)\right)\sin(\theta(t)) - 0.75\left(\frac{d}{dt}\, \theta(t)\right)^2\cos(\theta(t))\right) = 0 \tag{5.1}$$

> $de5 := \frac{\partial}{\partial t}\, de2$

(5.2)

$$de5 := \left(\left(\frac{d^2}{dt^2} s(t)\right)\sin(\beta(t)) + 2\left(\frac{d}{dt} s(t)\right)\left(\frac{d}{dt}\beta(t)\right)\cos(\beta(t)) + s(t)\left(\frac{d^2}{dt^2}\beta(t)\right)\cos(\beta(t)) - s(t)\left(\frac{d}{dt}\beta(t)\right)^2 \sin(\beta(t)) = 0.75\left(\frac{d^2}{dt^2}\theta(t)\right)\cos(\theta(t)) - 0.75\left(\frac{d}{dt}\theta(t)\right)^2 \sin(\theta(t))\right) = 0 \quad \textbf{(5.2)}$$

> $de6 := \frac{d^2}{dt^2}\theta(t) = \alpha :$

> $res3 := dsolve(\{de4, de5, de6, \beta(0) = 0, s(0) = 3.25, \theta(0) = 0, D(\beta)(0) = 0, D(s)(0) = 0, D(\theta)(0) = 0\}, \{\beta(t), s(t), \theta(t)\}, numeric)$

$$res3 := \textbf{proc}(x\_rkf45) \ \dots \ \textbf{end proc} \quad \textbf{(5.3)}$$

> plots:-odeplot(res3, [t, beta(t)], t = 0 ..0.6, title = "Crank Angle vs Time")