Mathematical Experiments for Mathematics Majors

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MACM 204-2 Computing with Calculus using Maple

Prerequisites: Integral Calculus and Programming 1.

Goals

1. Practice programming skills on mathematics problems.
2. Review Calculus and apply it to realistic problems.
3. Use visualization tools to understand/present mathematics.
4. **Learn to do mathematical experiments.**
5. Master the software package for use in other courses/career.
6. Modelling (systems of ODEs) e.g. the SIR model.

Six experiments, one per assignment, requires programming.

1: Checking theorems

Theorem $\int_{-\pi/2}^{\pi/2} \sin(mx)\cos(nx)dx = 0$ for all nonnegative integers $m > 0$ and $n > 0$.

(a) Use Maple to check it for $1 \le m \le 5$ and $1 \le n \le 5$.

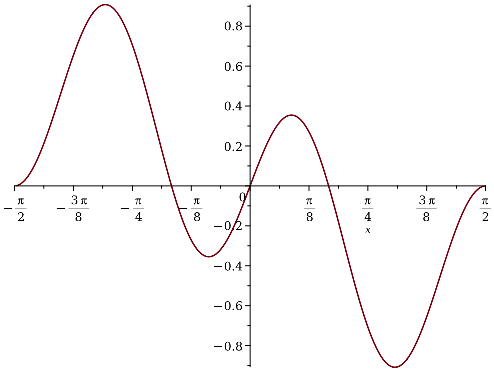
```
> for m from 1 to 5 do
>     for n from 1 to 5 do
>         print(m,n,int(sin(m*x)*cos(n*x),x=-Pi/2..Pi/2));
>     od;
> od;
```

```
1, 1, 0
1, 2, 0
1, 3, 0
  ...
5, 5, 0
```

I found this unsatisfactory because ...

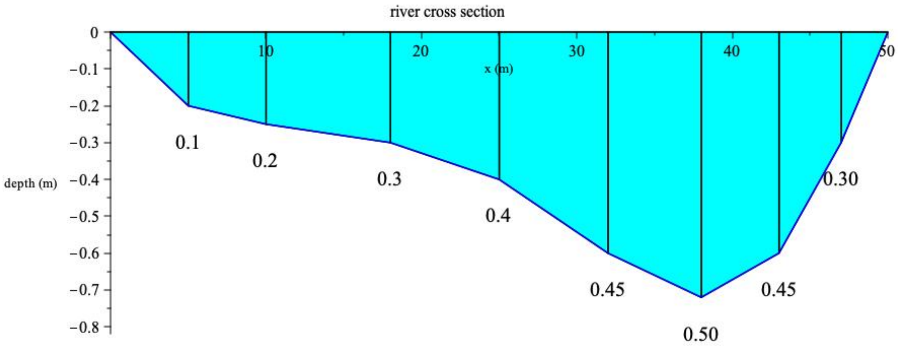
(b) Now graph $\sin(mx)\cos(nx)$ on $-\pi/2 \le x \le \pi/2$ for $m = 2, 3$ and $n = 2, 3$ and explain why they are orthogonal.

```
> plot(sin(3*x)*cos(2*x),x=-Pi/2..Pi/2 );
```

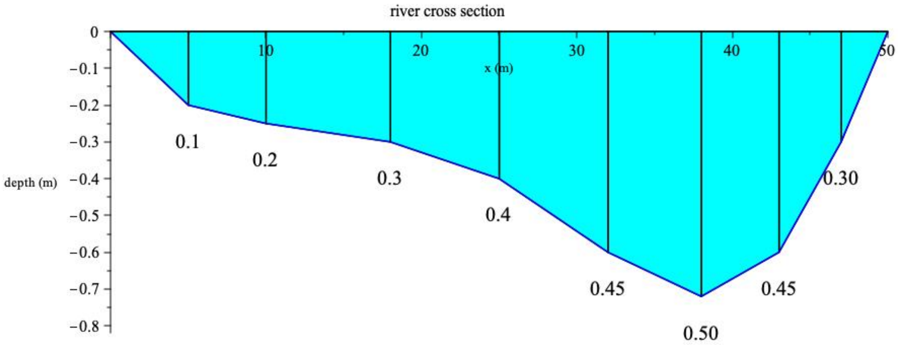


The integral is zero because $\sin(mx)\cos(nx)$ simplifes to a sum of sine functions, which are odd. The integral of an odd function over a symmetric interval $[-\pi, \pi]$ is always zero.

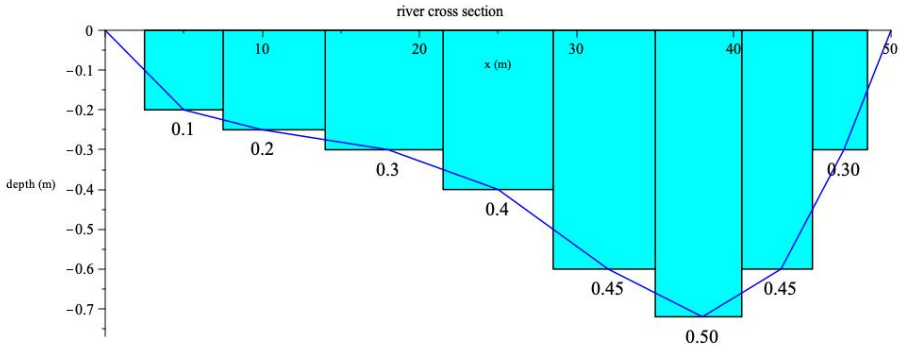
3: Measuring river flow (discharge)



distance x m	0	5	10	18	25	32	38	43	47	50
depth d m	0.0	0.2	0.25	0.3	0.4	0.6	0.72	0.6	0.3	0.0
velocity v m/s	0.0	0.1	0.2	0.3	0.4	0.45	0.5	0.45	0.30	0.0



Method 1: Using trapezoids and velocity averages



Method 2: Using midpoint rectangles and velocites

Which approximation is best?

Flow $= \int_a^b v(x)d(x)dx \ m^3/s$ where $d(x)$ is the depth and $v(x)$ is the velocity at x.

2 Prime number races (Granville and Martin)

4 Random walks in the plane

5 Visualizing eigenvalues and eigenvectors

6 Simulating a 30 year mortgage

7 Simulating vaccination in the SIR model to determine herd immunity

Questions