From
2600 BC

https://www.peruforless.com/blog/quipu

Quipu: Ancient Computer of the Inca Civilization

How Will Mathematics Be Learned in the Age of AI?

Deborah Hughes Hallett
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Extrapolating from History

- **History**: How have previous technological advances affected mathematics?
- **Peering into the Future**: What does the past tell us is likely to happen now?

What was happening 165 years ago?

Harvard Exam

Question Five: Math (1860)

"What is the square root of 104.8576?"

What was the point of this question?

https://www.thecrimson.com/flyby/article/2010/12/8/harvard-exams-question-1860/

In the days BC:

(Before Calculator)

How did you

- Multiply 125 x 38.27?
- Divide 125/38.27?

What did scientists do? What did students do on homework and exams?

Calculation was drudgery…..lead to "sanitized" applications. Most problems were restricted to doing a calculation

What is this ???? What is it for?

A *Slide Rule*, for multiplication and division of "messy" numbers

How did it work? By adding and subtracting lengths that corresponded to the numbers

The lengths are (proportional to) logarithms

http://www.hpmuseum.org/versa2.jpg

The Four-Function Calculator

Early 1970s: Calculators and Computers

Concerns:

- **What will happen to students' ability to do arithmetic?**
- They will not know their times tables; they will not be able to estimate the magnitude of an answer
- These abilities will (and did) wither. Was that a disaster?

Outcomes:

- No, it **shifted the focus away from pure computation and toward the meaning of computations** (that is, toward concepts and applications).

Scientific Calculator

Concerns:

- Log tables gone for good!
- For better or worse, **arithmetical computation was no longer a barrier to entry in mathematics.**

Pulling up the drawbridge:

- Don’t allow calculators on tests.

https://www.target.com/p/

Calculation to Communicate Numbers

Annual vehicle deaths in US: in 2024 there were 39,345 deaths

- **How many per day? Per hour? 110 per day or 4 per hour**

$$\frac{39345}{366} \approx 110 \text{ or } \frac{110}{24} \approx 4$$

What is the present value of $1 million in 40 years at a discount rate of 3%? 6%?

- **At 3%,**

$$PV = \frac{10^6}{(1.03)^{40}} = 306,556.8 \approx \$300,000$$

- **At 6%,**

$$PV = \frac{10^6}{(1.06)^{40}} = 97,222.8 \approx \$97,000$$

Doubling the discount rate reduces PV to a third.

https://en.wikipedia.org/wiki/Motor_vehicle_fatality_rate_in_U.S._by_year

Graphing Calculators

"The Disk with the College Education" 1982
"A Calculator with a College Education" 1987

- Before graphing calculators emerged, **graphing a function was an end in itself**, not a tool to illustrate other questions.
- Graphing calculators turned that on its head.
- Again, there were predictions of disaster, and efforts to ban them. Currently **the Advanced Placement allows them on parts of exams**.
- As earlier, **graphing calculators shifted the focus**: This time away from creating the graph toward interpreting the graph.

AMS Monthly, H. Wilf 1982
Calculus for a New Century, T. Tucker 1987
en.wikipedia.org/wiki/TI-81

If $f(x) = x^3 + x + 1$.
Estimate $f^{-1}(20)$?

About 2.5

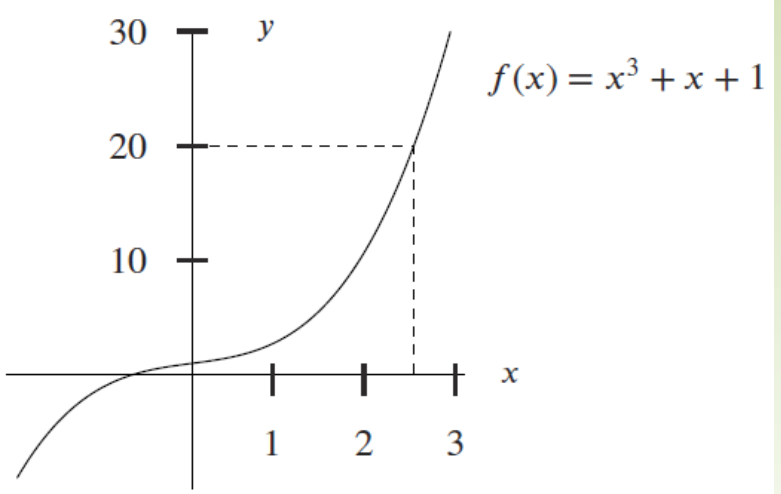


Figure 9

For what values of x is $4^x > x^4$?

For $-0.8 < x < 2, x > 4$

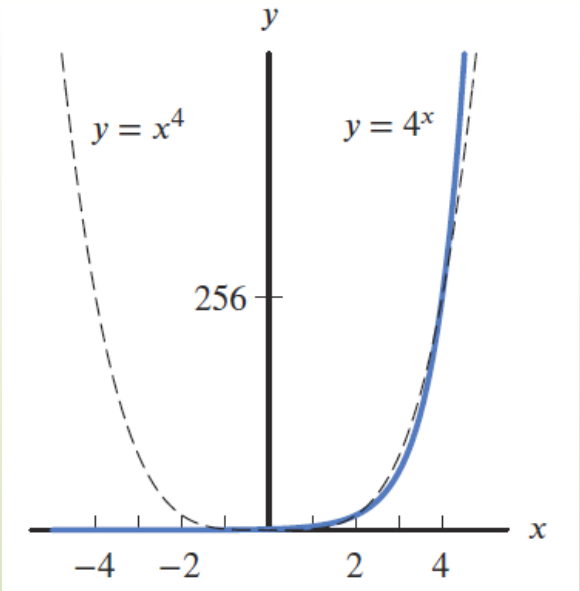


Figure 4: Solid $y = 4^x$; dashed $y = x^4$

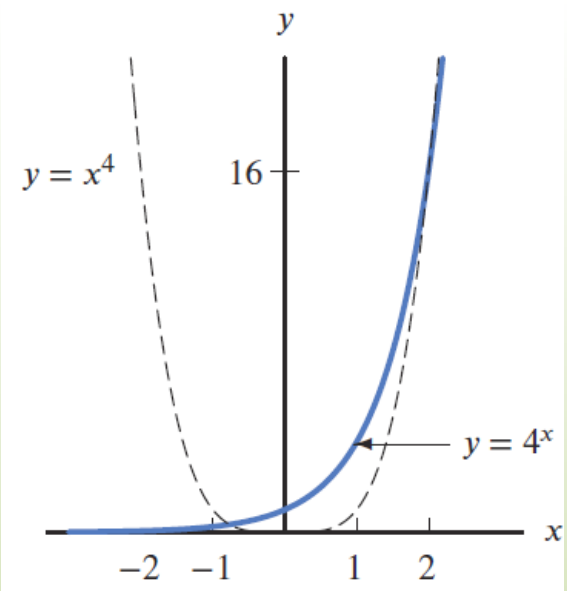
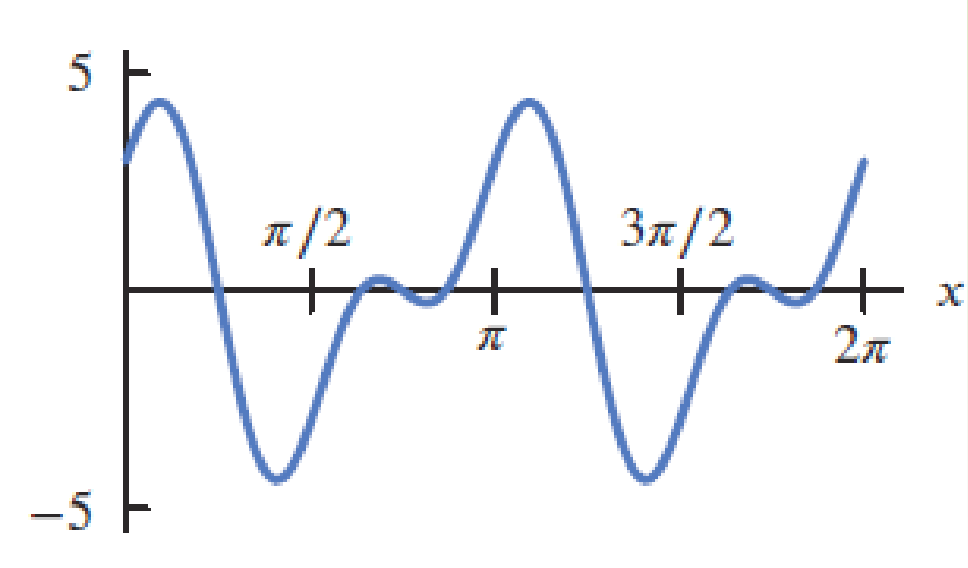


Figure 5: Close up

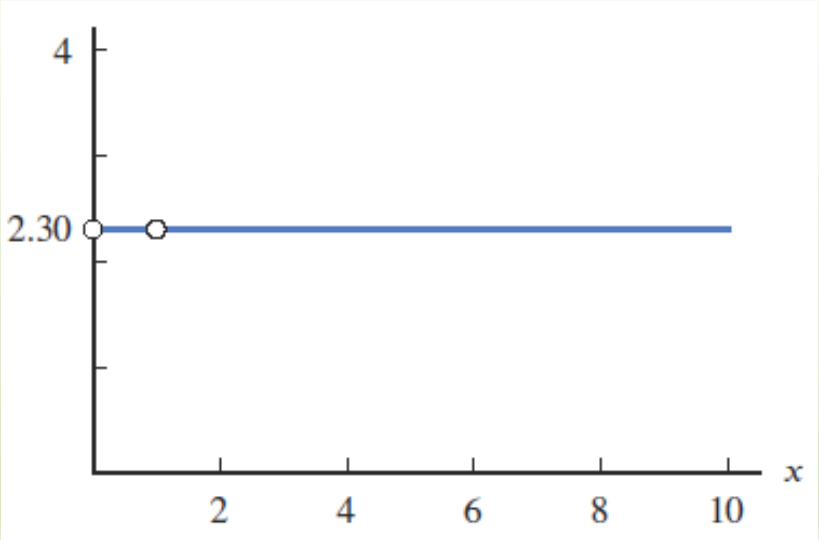
From *Calculus,* D. Hughes Hallett, Wiley

What is the period of $2\sin 4x + 3\cos 2x$?



Answer: π

Plot $(\ln x)/(\log x)$ Explain what you see.

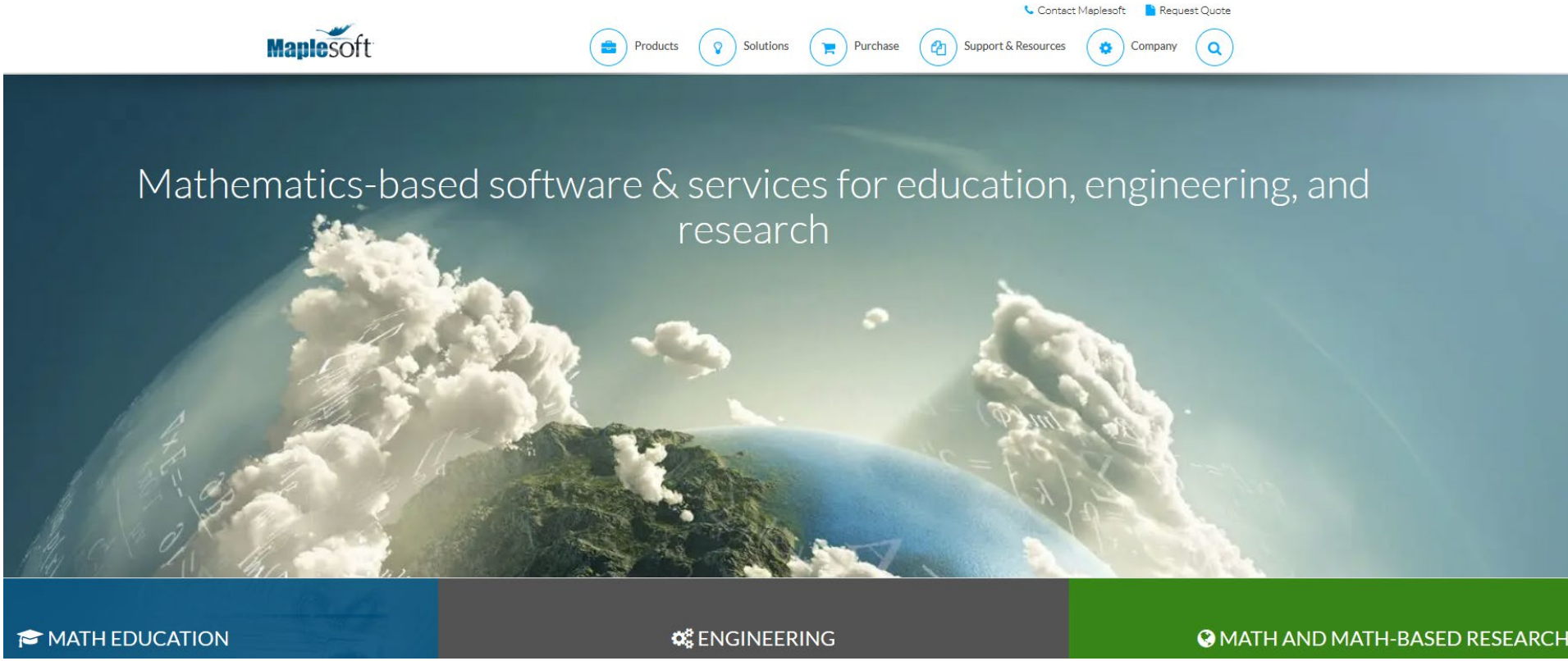


From *Calculus,* D. Hughes Hallett, Wiley

Computer Algebra (CAS)

20 April 1988 https://www.maplesoft.com/

TI Nspire 2007



Computer Algebra (CAS)

- **CAS were important to STEM and research** long before they affected education.
- Coming of CAS calculators changed that.
- Again, a **shift is taking place**, toward interpretation of symbolic form and away from manipulation of symbolic form.
- Here we are in the middle of the shift, so CAS are not allowed on many exams. (No QWERTY keyboard on Advanced Placement. There are two versions of TI Nspire, one with CAS and one without)

Maple

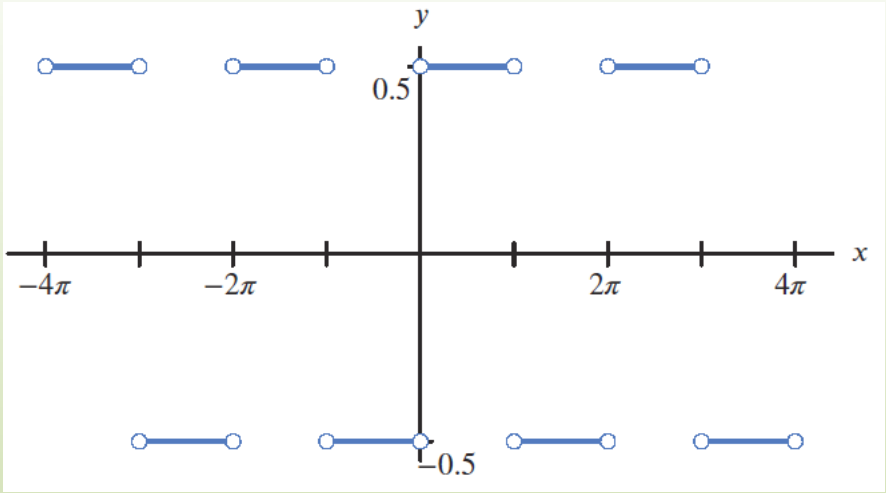
Student Success

TI Nspire

Use a CAS to find the derivatives of $\ln x$, $\ln(2x)$, $\ln(3x)$. Explain.

All derivatives are $^1/_x$

Find and graph the derivative of $y = \tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right)$



From *Calculus,* D. Hughes Hallett, Wiley

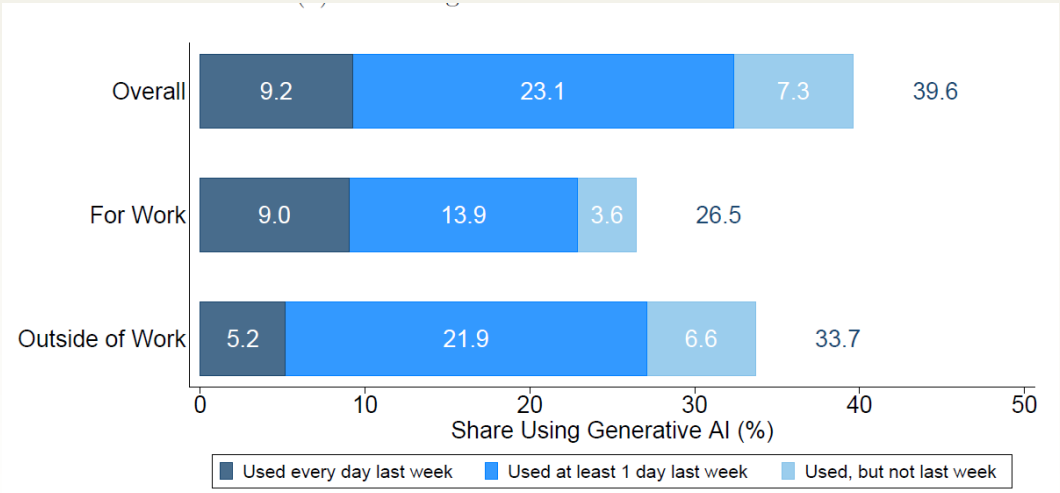
Looking Back at Each Innovation

- **What happened initially?**
  - Some excitement
  - Widespread pushback
  - Concern that basic skills would be lost
  - Skills under threat had previously been a central focus
- **Then what?**
  - The fact that the skill is one that can be done by technology makes students---and later instructors---think twice about its value
  - Triggers re-evaluation of basic skills
- **Shift towards asking different questions**
  - Usually new questions are ones that take the previously taught procedures as given and asks interpretive questions

To Intervene, First Understand?

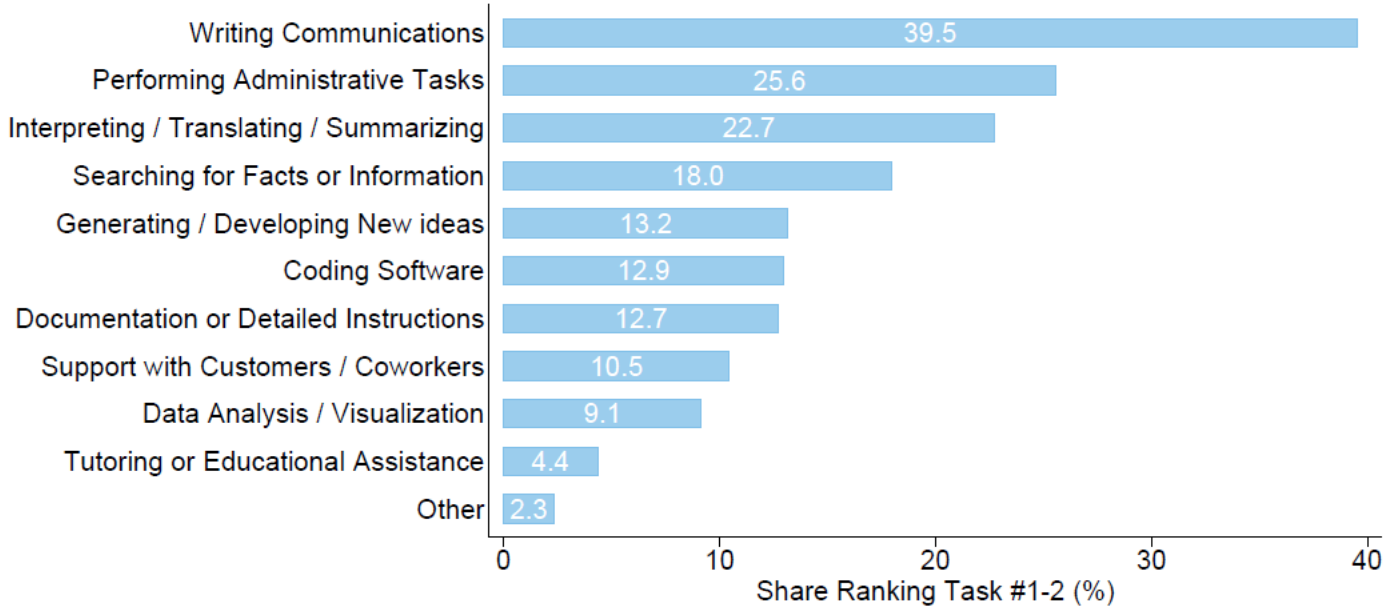
- **What is AI currently being used for?**
  - Use LLMs to write emails, analyze data, summarize reports, draft memos, and so on.
  - Making workers in entry level positions more efficient.
- **How do students and professionals learn this?**
  - By trying things out
  - Researching a topic, designing a company or NGO, finding a hook for a marketing campaign, writing R or python code, analyzing spreadsheet data, making graphs.

Current AI Usage: Nov 2024



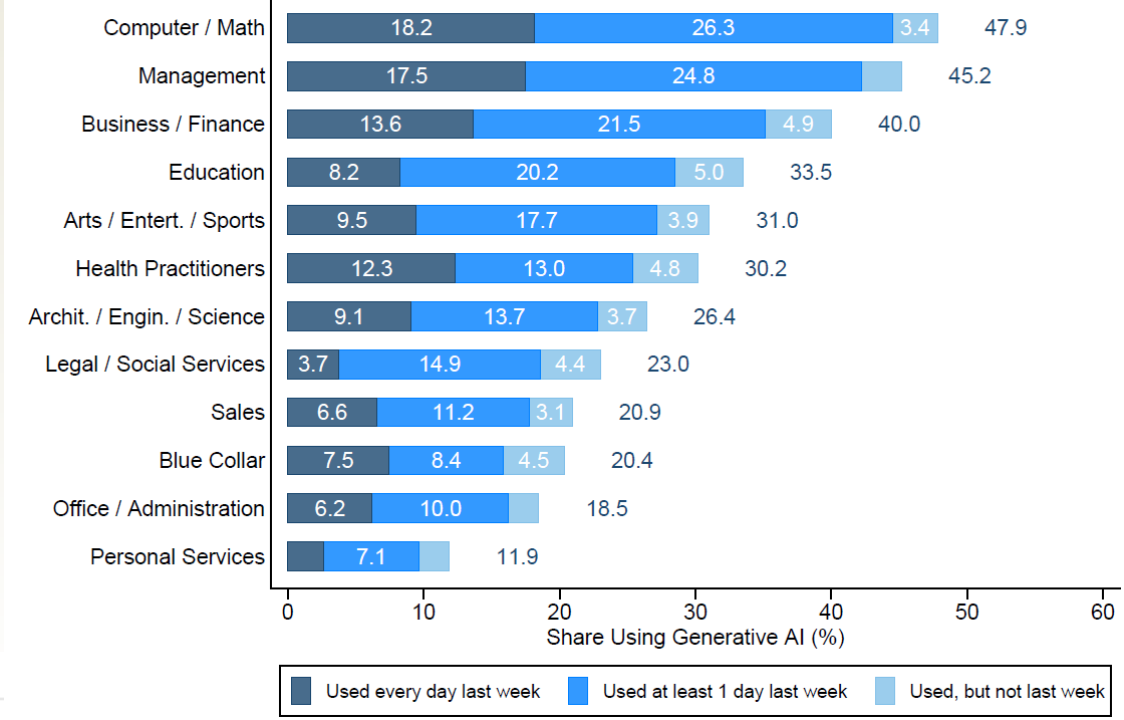
Working Adults 18-64

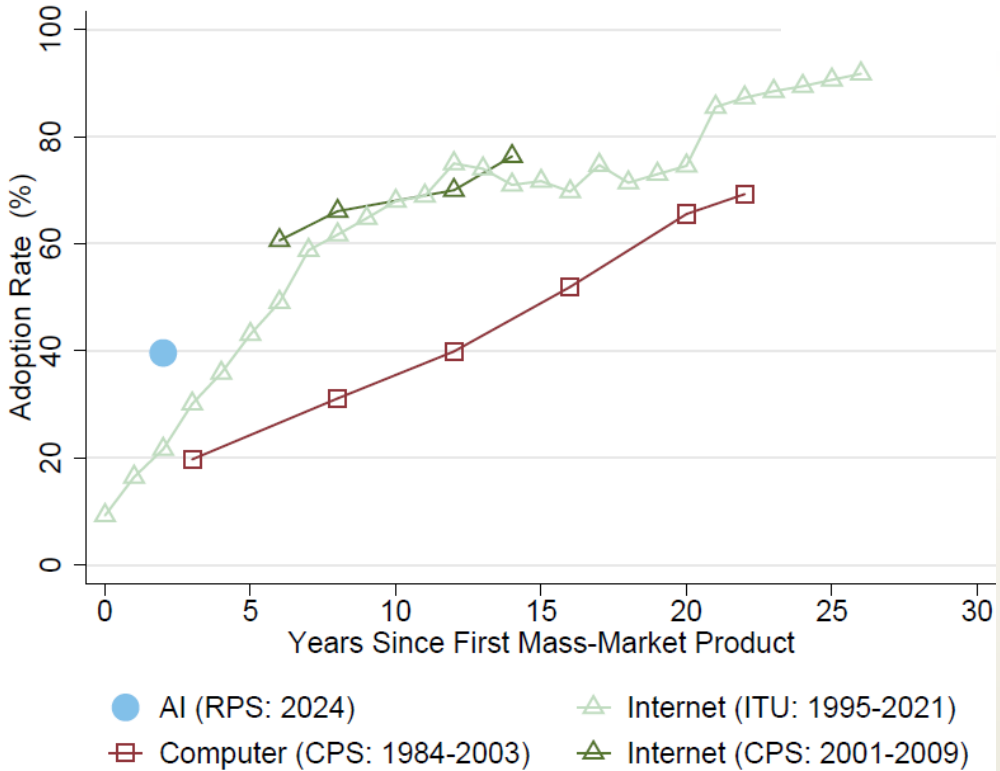
What for?



"The Rapid Adoption of Generative AI", A. Bick, A. Blandin, D. Deming, Sept 2024

Who and How Fast?



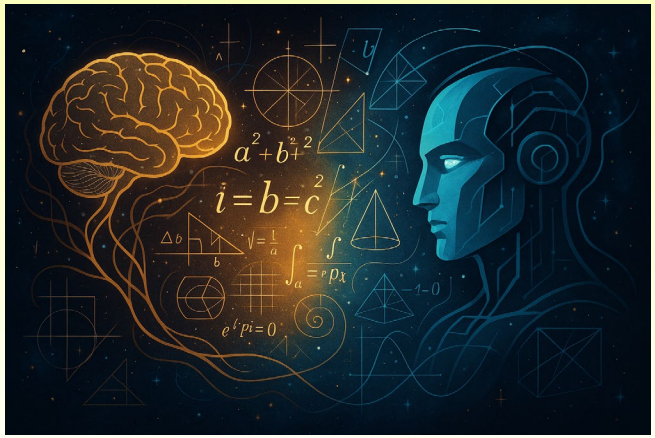


"The Rapid Adoption of Generative AI", A. Bick, A. Blandin, D. Deming, Sept 2024

Where Are We?

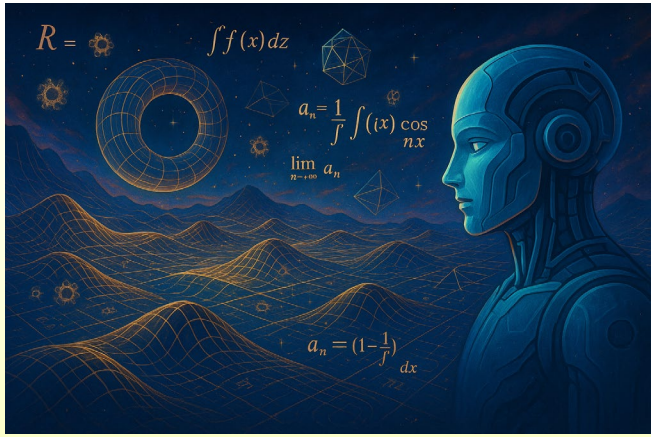
- Currently, even with CAS, students have to think about what method to use (unless problem specifies it)
- Now, AI will tell them.
- Students still need to check that the output is right
- *What do they need to know to check that the output is correct*?
- **Most Important: Learning to ask good questions**

What Do You Want in New Problems?



They should involve

- Reasoning, not calculation
- Comparing models, arguments, assumptions
- Partnering between student and AI



How long will they be useful?

- Anyone's guess!
- AI can already do this reasoning
- AI is changing fast!

Pairings: Gen AI and Maple

- Pairing: **Use genAI to write Maple code**.
  - Doesn't solve problem that people go to genAI first
- Pairing: **Use Maple to check genAI output**
  - Accurate and known to be accurate. No hallucinations!
- **Challenge**: What can Maplesoft do that genAI cannot?

Thank you!

What questions/comments do you have?

Gen AI and Large Language Models

First released November 2022

GPT-4.1

Area	Before LLMs	After LLMs (e.g., GPT-4.1)
Problem-solving	Manual, stepwise, time-intensive	Automated, fast, step-by-step with explanation
Homework Integrity	Cheating requires effort	Cheating is easy and automated
Access to Help	Dependent on teacher/tutor	Immediate, 24/7 personalized help
Classroom focus	Procedures, calculations	Interpretation, problem pose, critique
Teacher workload	High (manual grading, prep)	Reduced (automated grading, resource creation)
Learning Equity	Depends on resources	Potentially increased, but not guaranteed

- **Math skills still matter,** but the emphasis will be on sense-making, creativity, and critical engagement—not rote calculation or replicated solution patterns.
- **Educators who embrace LLMs** as tools for exploration, communication, and assessment innovation will help students thrive in the AI era.
- **Ongoing adaptation** is essential, as LLMs (and their symbolic/visual assistants) will become ever more capable.

1. The “AI Got It Wrong” Problem

Topic: Calculus – Optimization
Prompt:
Ask ChatGPT to solve this problem:
“A rectangle is inscribed under the parabola (y = 12 - x^2) and above the x-axis. Find the dimensions of the rectangle with maximum area.”
Your task:
1.Record ChatGPT’s full solution.
2.Identify any assumptions or errors in its reasoning (if any).
3.Explain *why* the correct maximum occurs at the point you claim, using derivative reasoning and a clear argument.
4.Reflect: what makes this problem tricky for an AI or for a human?
Why it works:
Students must *evaluate* and *defend* reasoning, not just copy it. Even if ChatGPT is right, they have to justify *why* it’s right.

2. The “Multiple Models, One Result” Problem

Topic: Algebra / Precalculus – Modeling & Functions
Prompt:
The population of a bacteria colony doubles every 3 hours for the first 12 hours, then starts slowing down because of limited nutrients.
1.Create **two different models** (one exponential, one logistic) that fit this situation.
2.Use ChatGPT to generate each model and its predictions.
3.Compare and critique: which model better captures the biology of the system, and why?
4.Discuss what assumptions about the real world each model makes.
Why it works:
Students must interpret and judge *modeling assumptions* and fit, not just generate formulas.

3. The “AI as a Partner in Proof” Problem

Topic: Geometry or Linear Algebra – Proof and Reasoning

Prompt:

Use ChatGPT to help you explore this conjecture:

“The diagonals of a parallelogram bisect each other.”

1. Ask ChatGPT to produce a formal proof.
2. Analyze its proof: is it logically valid? Does it rely on unstated assumptions?
3. Rewrite the proof in your own words, making it more rigorous or more intuitive.
4. Extend the question: does this property still hold in 3-dimensional space for a *parallelepiped*? Explain or give a counterexample.

Why it works:

Students must *critically evaluate* an AI-generated proof, learn formal structure, and extend reasoning beyond what the model provides.